# VASAVI COLLEGE OF ENGINEERING (Autonomous), HYDERABAD B.E. (CSE: CBCS) MI-Semester Supplementary Examinations, May/June-2018 

## Discrete Structures

Time: $\mathbf{3}$ hours
Max.Marks: 70
Note: Answer ALL questions in Part-A and any FIVE from Part-B
Part-A $(10 \times 2=20 \mathrm{Marks})$

1. Write converse, contrapositive and inverse of the conditional proposition $p \rightarrow q$.
2. List out any four English constructs of biconditional connective $p \leftrightarrow q$.
3. Define an injective function and give an example.
4. Let $f: \mathrm{R} \rightarrow \mathrm{R}$ be given by $f(x)=\mathrm{x}^{3}-2_{\imath}$, Find $f^{-1}$.
5. Solve the recurrence relation $F_{n}=F_{n-1}+F_{n-2}$, where $F_{0}=0, F_{1}=1$
6. Write the Generating function that generates sequence $1,4,9,16,25, \ldots \ldots \ldots \ldots$
7. Determine whether each of the following pairs of integers is congruent modulo 9
i) $-137,700$
ii) $-56,-1199$.
8. Define "multiplicative inverse" and give an example
9. What is an Hamming Metric ?
10. Define a monoid.

$$
\text { Part-B }(5 \times 10=50 \text { Marks })
$$

11. a) Use substitution rules to verify that each of the following is a tautology.(Here $\mathrm{p}, \mathrm{q}$ and $r$ are primitive statements).
a) $[p \vee(q \wedge r)] \vee \square[p \vee(q \wedge r)]$
b) $[(\mathrm{p} \vee \mathrm{q}) \rightarrow \mathrm{r}] \leftrightarrow[\neg \mathrm{r} \rightarrow \neg(\mathrm{p} \vee \mathrm{q})]$

Verify Absorption Laws by means of a truth table.
12. a) Draw the Hasse diagram of the following sets under the partial ordering relation "divides," and indicate those which are totally ordered.
$\{2,4,8,16\} \quad\{1,2,3,6,12\} \quad\{3,5,15\}$
b) Let $X=\{1,2, \ldots \ldots .7\}$ and $R=\{(x, y) \mid x-y$ is divisible by 3$\}$.

Show that $R$ is an equivalence relation.
13. a) Solve the recurrence relation $a_{n}-8 a_{n-1}+16 a_{n-2}=8(5)^{n}$ where $a_{0}=12, a_{1}=5$
b) In how many ways can two dozen identical robots be assigned to four assembly lines with i)at least three robots assigned to each line? ii) at least three, but no more than nine robots to each line?
14. a) Let $(\mathrm{R},+$, . ) be a ring with $\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}$ elements of R . State the conditions(from the definition of a ring) that are needed to prove each of the following results.
i) $(a+b)+c=b+(c+a)$
ii) $c(d+b)+a b=(a+c) b+c d$
b) If $f: R \rightarrow S$ is a ring homomorphism and $J$ is an ideal of $S$, prove that $f^{-1}(J)=\{a \in R \mid f(a) \in J\}$ is an ideal of $R$
15. a). In a monoid, show that the set of left-invertibles form a sub-monoid.
b) Prove that for any commutative monoid $(\mathrm{M}, *)$, the set of idempotent elements of $M$ forms a submonoid.
16. a) Prove that any $Z^{+}, \operatorname{gcd}(5 n+3,7 n+4)=1$.
b) Let $f: R->R$ is a function, $f(x)=3 x-5$ where $x>0, f(x)=-3 x+1$ where $x<=0$, then find $f(0), f(5 / 3), f^{-1}(-3), f^{-1}(6), f^{-1}([5,-5])$
17. Answer any two of the following:
a) Solve the following recurrence relation

$$
\begin{equation*}
a_{n+2}+4 a_{n+1}+4 a_{n}=7, n \geq 0, a_{0}=1, a_{1}=2 \tag{5}
\end{equation*}
$$

b) Determine the plain text for the RSA cipher text $\begin{array}{lllllll}1418 & 1436 & 2370 & 1102 & 1805\end{array}$ 0250 if $\mathrm{e}=11$ and $\mathrm{n}=2501$.
c) i) Let $\mathrm{p}=0.01$ be the probability of incorrect transmission for a binary symmetric probability of correct decoding? ii) Answer part (i) for a 20-bit message sent in five blocks of length 4 .

